PHYSICAL FINSLER COORDINATES FOR CLASSICAL MOTION

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It was argued in earlier work that the four-velocity of a measured quantum particle excitation of a Finslerian quantum field in the tangent space manifold of spacetime is not a suitable Finsler coordinate, whereas the four velocity of the measuring device relative to the vacuum is a suitable Finsler coordinate. Furthermore, in the present work, it is argued that the physical Finsler coordinate for describing the classical motion of a macroscopic object is the four-velocity of the classical object, which in effect acts as a measuring device measuring the characteristics of the metric field. Specifically, geodesic motion of a macroscopic object in a Finslerian spacetime is considered, where the appropriate physical Finsler coordinate is the four-velocity of the object undergoing geodesic motion. It is also claimed that for a macroscopic object, such as a macroscopic measuring device, consisting of more than Avogadro's number of atoms, any supposed quantum state is negligibly small, so that for all practical purposes the object is best described by classical mechanics. It is argued that this and the above follow from a reasonable upper bound on physically possible proper acceleration.

Key Words: Finslerian spacetime, Finsler coordinates, spacetime tangent bundle, maximal proper acceleration, geodesics, quantum-classical boundary, Avogadro's number.

1 Introduction

It was argued in earlier work that physically meaningful coordinates of a point in the tangent bundle of spacetime are the spacetime coordinates and the four-velocity coordinates of the measuring device relative to the vacuum [1, 2]. In the one case considered earlier, the moving device detected a relativistic vacuum particle excitation of a Finslerian quantum field. It was argued that the four-velocity of a measuring device is the physical Finsler coordinate, whereas the four-velocity of the measured quantum particle is not. The four-velocity of a measured quantum particle excitation of a Finslerian quantum field in the tangent space manifold of spacetime is not a suitable Finsler coordinate. This is related to the fact that, because of the quantum uncertainty principle, the particle velocity at a point in spacetime is intrinsically unknown.

In the present work, I argue that the physical Finsler coordinate for describing the classical motion of a macroscopic object is the four-velocity of the classical object with respect to the metric field. The object effectively acts as a measuring device measuring the metric field. In the case of standard classical electrodynamics, for example, the Lorentz force depends explicitly on the four-velocity of a classical charged particle with respect to the electromagnetic field and can serve as a measuring device in which its motion is effectively a measure of the strength of the electromagnetic field acting on the classical particle. Also, in the case of the classical Vlasov equation, including a gravitational field, the Finsler coordinates are the four-velocities of the classical particles in the Vlasov distribution of particles moving in spacetime, and the Vlasov distribution effectively measures the spacetime metric field [3], [4].

In the present work, I consider geodesic motion of a macroscopic object in a Finsler-spacetime tangent bundle, incorporating the constraint of a limiting proper acceleration [5],

[1]Dedicated to the distinguished Finsler geometer, Prof. Dr. Lajos Tamassy, on his 90th birthday

in which the appropriate physical Finsler coordinate is the four-velocity of the object undergoing geodesic motion. The motion of the classical object effectively measures the presence of the spacetime metric field because, in the absence of non-gravitational forces, the motion of classical objects is determined by the spacetime metric through its appearance in the geodesic equation of motion [6]. It is significant to note that because the spacetime metric depends on the Finsler coordinate, the restriction of the four-velocity to the four-velocity shell may require, in general, a multi-sheeted structure [7].

It is also argued, on the basis of the Finsler coordinate and the associated maximal proper acceleration, that for macroscopic objects (including measuring devices) consisting of more than Avogadro's number of atoms, any supposed quantum state of the object is negligibly small, so that, for all practical purposes, the object is best described by classical relativistic mechanics.

2 Classical particle geodesic motion

Classical particle geodesic motion in the tangent bundle of spacetime with the constraint of a limiting proper acceleration was exposited in earlier work [6]. In the simple case of a Riemannian-spacetime base manifold, it was argued that the natural lift of a classical particle geodesic in spacetime is also geodesic in the tangent bundle of spacetime. In this case the geodesic motion in the tangent bundle of spacetime is given generally by [6]:

$$\frac{D^2x^\mu}{d\sigma^2}+\frac{c^4}{a_0^2}v^\lambda R_{\alpha\lambda\beta}{}^{\mu}\frac{Dv^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}=0, \tag{1}$$

and

$$\frac{D^2v^\mu}{d\sigma^2}=0, \tag{2}$$

in which D denotes a covariant differential, $d\sigma$ is the invariant infinitesimal path length along the particle trajectory in the spacetime tangent bundle, namely,

$$d\sigma^2=\left(1-\frac{a^2}{a_0^2}\right)ds^2, \tag{3}$$

where ds is the ordinary spacetime line element, a is the proper acceleration of the classical particle, a_0 is the limiting proper acceleration [8], c is the speed of light in vacuum, and $R^\alpha{}_{\lambda\mu\nu}$ is the spacetime curvature tensor. Here, it is to be emphasized that v^μ in Eqs. (1) and (2) is the Finsler coordinate, namely, the four-velocity of the classical particle relative to the frame in which the metric field is defined. It is reasonable to interpret the classical particle as a measuring device whose motion effectively measures the strength of the metric field as it appears in the curvature and connection. Thus v^μ is effectively the four-velocity of the measuring device, namely, if x^μ denotes the spacetime coordinates of the device, then

$$v^\mu=\frac{dx^\mu}{ds}, \tag{4}$$

as was also the case in the analysis of quantum fields in the tangent bundle of spacetime [1], [2]. Also noteworthy is that the Finsler coordinate v^μ must lie on the four-velocity shell, namely, for spacetime metric $g_{\mu\nu}$, one requires [7]

$$g_{\mu\nu}(x,v)v^\mu v^\nu=1, \tag{5}$$

which implies the possibility in a general Finslerian spacetime tangent bundle of a multi-sheeted structure consisting of multiple distributions in the fiber, associated with the respective roots of Eq. (5). Multivaluedness of the Finsler coordinate evidently corresponds to different possible trajectories of the measuring device in four-velocity space. These correspond to trajectories of the measuring device on different sections of the bundle. Any measuring deice lies at any instant not only in the neighborhood of a single point in spacetime but also in the neighborhood of a single point in four-velocity space. If the base manifold is Riemannian, this multivaluedness is not an issue because in this case, the spacetime metric is independent of the four-velocity coordinates.

In a canonical pure gravitational field, according to general relativity, one has in Riemannian spacetime the standard geodesic equation with vanishing four-acceleration in the absence of non-gravitational forces,

$$\frac{Dv^\mu}{ds} = \frac{dv^\mu}{ds} + \Gamma^\mu{}_{\alpha\beta} v^\alpha v^\beta = 0, \tag{6}$$

where $\Gamma^\mu{}_{\alpha\beta}$ is the Riemannian Levi-Civita connection. In this case the proper acceleration is vanishing, as it is in general relativity in the absence of non-gravitational forces, namely,

$$a^2 = -c^4 g_{\mu\nu} \frac{Dv^\mu}{ds} \frac{Dv^\nu}{ds} = 0, \tag{7}$$

and according to Eq. (3), the bundle line element collapses to the ordinary spacetime line element,

$$d\sigma^2 = \left(1 - \frac{a^2}{a_0^2}\right) ds^2 = ds^2, \tag{8}$$

or

$$\frac{d\sigma}{ds} = 1 \tag{9}$$

for the spacetime geodesic, Eq. (6). It then follows from Eqs. (9) and (6) that

$$\frac{Dv^\mu}{d\sigma} = \frac{Dv^\mu}{ds} = 0 \tag{10}$$

for the geodesic lift. Substituting Eqs. (10) and (4) in Eqs. (1)-(2), it follows that the geodesic natural lift trivially satisfies

$$\frac{Dv^\mu}{ds} + \frac{c^4}{a_0^2} R_{\alpha\lambda\beta}{}^{\mu} v^\lambda v^\beta \frac{Dv^\alpha}{ds} = 0, \tag{11}$$

and

$$\frac{D^2 v^\mu}{ds^2} = 0. \tag{12}$$

Thus according to Eq. (10), it follows that Eqs. (11) and (12), are both satisfied for a spacetime geodesic. One concludes that if the spacetime is Riemannian, then the natural lift of a spacetime geodesic, representing the path of the measuring device, is also a geodesic in the spacetime tangent bundle. However it is important to note that for non-Riemannian spacetimes (e.g. Finsler spacetime), any relation between geodesics in the bundle and in spacetime will generally be very complex.

The role of the Finsler coordinate as the four-velocity of a measuring device was also manifest in an earlier analysis of the intrinsic redshift of star light as measured at large distances, resulting from enforcing the constraint of a limiting proper acceleration, leading to a perturbed Schwarzschild solution for the metric outside a star [9], [7].

From the above, one can conclude that a measuring device for measuring both quantum and classical fields is generally a macroscopic object. In the following it is argued by means of a simplified quantum field theoretical model that a macroscopic object, such as a measuring device, is well described by classical mechanics, as is the case in the standard quantum mechanics of measurement in which the measuring device is treated as a classical macroscopic device [11].

3 The quantum-classical boundary

There is no generally accepted theory of why the world of macroscopic objects is not usefully described in terms of quantum states. For example, a planet is never observed to be in a quantum superposition state. It has however been speculated that for the description of classical macroscopic many-body systems of sufficient complexity, quantified by the number of atoms of which it is composed, quantum mechanics can be replaced by classical mechanics. (Of course for a highly correlated mesoscopic system such as a Bose condensate, which consists of a single quantum state, a quantum description is needed. Also, quantum mechanics is clearly needed to understand the atomic and molecular structure of macroscopic objects) It is here to be shown that for macroscopic objects consisting of more than Avogadro's number of atoms, any supposed quantum state of the object as a whole is negligibly small, so that for all practical purposes the system is best described by classical mechanics. This follows from the physics-based upper bound on physically possible proper acceleration [8–10].

A possible implication of the limiting proper acceleration a_0 is that for a free particle with four-momentum p^μ and for a measuring device with spacetime coordinates x^μ and four-velocity coordinates v^μ with respect to a particle excitation in flat Minkowski spacetime, the quantum state is given by [12–14]

$$\psi(x,v) = \langle 0|\phi(x,v)|p\rangle \,, \tag{13}$$

in which $|0\rangle$ is the vacuum state, $|p\rangle$ is the state of the particle with four-momentum p^μ, and the scalar quantum field associated with the particle is given by

$$\begin{aligned}\phi(x,v) = 2\int \frac{d^3\mathbf{p}}{(2\pi\hbar)^{3/2}\,(2p^0N)^{1/2}}\,&\left[e^{-ipx/\hbar}e^{-\rho_0 pv/\hbar}\theta(\rho_0 pv/\hbar)a(\mathbf{p})\right.\\ &+\left. e^{ipx/\hbar}e^{\rho_0 pv/\hbar}\theta(-\rho_0 pv/\hbar)a^\dagger(\mathbf{p})\right].\end{aligned} \tag{14}$$

Here θ is the Heaviside step function; $\hbar$ is Planck's constant divided by 2π; N is a normalization constant; $a^\dagger(\mathbf{p})$ and and $a(\mathbf{p})$ are particle creation and annihilation operators for the spatial component of four-momentum $\mathbf{p}$, and ρ_0 is of the order of the Planck length, namely,

$$\rho_0 = \frac{c^2}{a_0}, \tag{15}$$

in which c is the speed of light in vacuum, and the limiting proper acceleration a_0 is given by

$$a_0 = 2\pi\alpha\left(\frac{c^7}{\hbar G}\right)^{1/2}, \tag{16}$$

where α is a number of order unity, and G is the universal gravitational constant. One notes that in the mathematical limit of infinite maximal proper acceleration a_0, one has vanishing ρ_0, and Eq. (14) the reduces to the same form as a canonical Lorentz-invariant scalar quantum field (as it must).

It can be shown that both the positive and negative frequency terms in Eq. (14) and appearing in the quantum state Eq. 13) are proportional to [6, 14, 15]

$$\exp(-\rho_0|pv|/\hbar) = \exp\left[-\frac{1}{2\pi\alpha}\frac{\gamma m}{m_{Pl}}\left\{\left(1+\left|\frac{\vec{p}}{mc}\right|^2\right)^{1/2} - \frac{\vec{p}\cdot d\vec{x}/dt}{mc^2}\right\}\right], \tag{17}$$

where m is the rest mass of the quantum particle, m_{Pl} is the Planck mass, and

$$\gamma = \left(1-\left|\frac{d\vec{x}/dt}{c}\right|^2\right)^{-1/2}, \tag{18}$$

where $d\overrightarrow{x}/dt$ is the spatial component of the four-velocity of the measuring device relative to the particle. For velocities of the measuring device much less than the velocity of light, and for particles masses much less than the Planck mass, Eq. (17) is for all practical purposes unity because ρ_0 is so small (of the order of the Planck length).

From Eqs. (13)–(18), it follows that the quantum state, in the case in which the measuring device is at rest with respect to a particle excitation from the vacuum is given by

$$\psi(x, d\overrightarrow{x}/dt = 0) = A \exp\left(-\frac{1}{2\pi\alpha}\frac{m}{m_{Pl}}\left(1 + \left|\frac{\overrightarrow{p}}{mc}\right|^2\right)^{1/2}\right) \exp\left(-ipx/\hbar\right), \tag{19}$$

where A is a normalization constant. Equation (19) can be expected to hold for any nonrelativistic bosonic or fermionic state. Then suppose that this state (wave function) is taken to describe a macroscopic object containing more than Avogadro's number of atoms, in which case its mass m satisfies the following:

$$m > N_A m_n, \tag{20}$$

where N_A is Avogadro's Number, and m_n is the mass of a nucleon. Then according to Eqs. (19) and (20), one obtains for the quantum state of this macroscopic many-body object in this simplified model:

$$\psi < A' \exp\left(-\frac{1}{2\pi\alpha} N_A \frac{m_n}{m_{Pl}}\left(1 + \left|\frac{\overrightarrow{p}}{mc}\right|^2\right)^{1/2}\right) \exp\left(-iN_A px/\hbar\right), \tag{21}$$

in which A' is the normalization constant for N_A particles. Equivalently, one can write the state as a product of N_A copies of Eq. (19), since the exponents add. Substituting $N_A = 6 \times 10^{23}$, $m_n = 1.7 \times 10^{-27}$kg , and $m_{Pl} = 2.2 \times 10^{-8}$kg in Eq. (21), the many-body wave function is seen to be negligible. This suggests that, for all practical purposes, a macroscopic object, such as a macroscopic measuring device, should not be described by quantum mechanics, and instead is best described by classical mechanics.

4 Conclusion

It can be reasonably concluded that in both classical and quantum mechanics, the appropriate physically meaningful Finsler coordinate is the four-velocity of the measuring device relative to that which is being measured. Also, it is argued that the measuring device can be treated as a macroscopic many-body system described by classical mechanics.

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ФИНСЛЕРОВЫ ФИЗИЧЕСКИЕ КООРДИНАТЫ ДЛЯ КЛАССИЧЕСКОГО ДВИЖЕНИЯ

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Как было показано ранее 4-скорость возбуждения квантовой частицы финслерова квантового поля в касательном многообразии пространства-времени не является подходящей финслеровой координатой, в то время как 4-скорость измерительного прибора по отношению к вакууму является таковой. Более того, в настоящей работе показывается, что физические финслеровы координаты для описания классического движения макроскопического объекта – это 4-скорость классического объекта, которая в действительности выступает, как измерительный прибор, измеряющий характеристики метрического поля. В частности, рассмотрено движение по геодезической макроскопического объекта в финслеровом пространстве-времени, где подходящими финслеровыми координатами является 4-скорость объекта, лежащая в основании движения по геодезической. Также утверждается, что для макроскопического объекта, такого как макроскопический измерительный прибор, состоящий из атомов число которых превышает число Авогадро любое мыслимое квантовое состояние является пренебрежимо малым и, поэтому, для любых практических целей такой объект лучше описывается классической механикой. Отмечается, что все вышесказанное следует из разумной верхней границы на физически возможное релятивистски равноускоренное движение.

Ключевые слова: финслерово пространство-время, финслеровы координаты, касательное расслоение пространства-времени, максимально возможное релятивистски равноускоренное движение,квантово-классические границы, число Авогадро.