FINSLER GEOMETRY IN GTR IN THE PRESENCE OF A VELOCITY DEPENDENT GAUGE FIELD

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Fields around their source determine a curved geometry. Velocity dependent phenomena in these fields involve a curvature tensor, whose elements depend on the value and the direction of the velocity of the source of the interaction gauge field, as observed from the reference frame of the matter field. This double (space-time and velocity) dependency of the curvature requires the field to follow a Finsler geometry.

Key Words: fundamental interactions, field charges, invariance, covariance, isotopic field charge spin, conservation, symmetry, gauge boson, velocity dependent fields, Finslerian curvature tensor, modified Einstein equation.

1 Conservation laws in the presence of a velocity dependent gauge field

[1] showed that in the presence of a velocity dependent gauge field $D_{\dot{\mu}} = D_{\dot{\mu}}(\dot{x}^{\mu})$ with a Lagrangian density $L(\phi_k, D_{\dot{\mu},\alpha})$, where φ_k, $(k=1, \ldots, n)$ are the matter fields — which also includes the velocity field $\dot{x}^{\mu} = \dot{x}^{\mu}(x_{\nu})$, and $D_{\dot{\mu},\alpha}$, $(\alpha = 1, \ldots, N)$, are the (kinetic) gauge fields, assumed also that $L(\varphi_k, D_{\dot{\mu},\alpha})$ is invariant under the local transformations of a compact, simple Lie group G generated by T_{α}, $(\alpha = 1, \ldots, N)$, where $[T_{\alpha}, T_{\beta}] = iC^{\gamma}_{\alpha\beta} T_{\gamma}$, and $C^{\gamma}_{\alpha\beta}$ are the so-called structure constants, corresponding to the actually considered individual physical interactions symmetry group, there appear two conserved Noether currents:

$$J^{(1)\nu}_{\alpha} = \partial_{\mu} F^{(1)\mu\nu}_{\alpha} \quad \partial_{\nu} J^{(1)\nu}_{\alpha} = 0 \tag{1}$$

$$J^{(2)\nu}_{\alpha} = \partial_{\mu} F^{(2)\mu\nu}_{\alpha} \quad \partial_{\nu} J^{(2)\nu}_{\alpha} = 0. \tag{2}$$

These equations form a complete system with the additional condition

$$\frac{\partial L}{\partial(\partial_{\mu} D_{\dot{\nu},\alpha})} \partial_{\nu} \dot{x}^{\rho} + \frac{\partial L}{\partial(\partial_{\nu} D_{\dot{\mu},\alpha})} \partial_{\mu} \dot{x}^{\rho} = 0.$$

1.1 Mathematical background

[2] gave a mathematical proof for the conservation of the currents $J^{(1)\mu}_{\alpha}$ and $J^{(2)\nu}_{\alpha}$ as well as demonstrated that — at least in this specific case — the replacement of an $f(\dot{x}_{\mu}, x_{\nu})$ dependence with an $f(\dot{x}_{\mu}(x_{\nu}))$ dependence led to the same result. This can be seen easily, for the currents $J^{(1)\mu}_{\alpha}$ coincide with those what we received in a simply space-time dependent field. However, the introduction of a velocity dependent gauge field provided an additional $J^{(2)\nu}_{\alpha}$ current family that extends and coexists with the previous ones simultaneously. The extension of the arguments of the fields is in full agreement with the original general formulation of Noether's second theorem [3-5]. The simultaneous existence holds although the respective components of the two current families are not independent.

Application of Finsler geometry can be investigated by analysing the currents $J^{(2)\nu}_{\alpha}$ where the velocity dependence presents itself. $J^{(2)\nu}_{\alpha}$ which is a current interpreted in the velocity dependent gauge field, can be written in the form

$$J^{(2)\nu}_{\alpha}(x) = i\gimel \left[\frac{\partial L}{\partial(\partial_{\mu}\varphi_k)} (T_{\alpha})_{kl} \varphi_l(\dot{x}) \partial_{\mu} \dot{x}^{\nu} - C^{\gamma}_{\alpha\beta} D_{\dot{\omega},\beta}(\dot{x}) \partial_{\mu} \dot{x}^{\omega} \times F^{(2)\mu\nu}_{\gamma}(x) \right] \tag{3}$$

where ℷ [gimel, the third letter of the Hebrew alphabet] denotes a general coupling constant, which can be replaced by a concrete coupling constant for each individual physical interaction, for example by g for gravity.

Writing $F_\alpha^{2\mu\nu}$ [1] in the left side of (3) considering (2) and writing the covariant derivative (denoted by careted $\hat{\partial}_\mu$) of $F_\alpha^{(2)\mu\nu}$ in the form

$$\hat{\partial}_\mu F_\alpha^{(2)\mu\nu}(x) = \partial_\mu F_\alpha^{(2)\mu\nu}(x) + i\gimel C^\gamma_{\alpha\beta} D_{\dot{\omega},\beta} \partial_\mu \dot{x}^\omega \times F_\gamma^{(2)\mu\nu}(x)$$

one gets

$$\hat{\partial}_\mu F_\alpha^{(2)\mu\nu}(x) = i\gimel \frac{\partial L}{\partial(\partial_\mu \varphi_k)} (T_\alpha)_{kl} \varphi_l(\dot{x}) \partial_\mu \dot{x}^\nu \tag{4}$$

This form has the advantage that the right side of the equation depends solely on the matter fields, and all dependencies on the gauge fields are separated in the left side. The velocity dependent (that means, direction dependent) curvature tensors appear also in the left side of the equation.

1.2 Physical considerations

The physical meaning of the couple of conserved currents is the following.

Eq. (1) can be written in the form:

$$\partial_\mu F_\alpha^{(1)\mu\nu}(\dot{x}) = i\gimel \frac{\partial L}{\partial(\partial_\nu \varphi_k)} (T_\alpha)_{kl} \varphi_l(\dot{x}) \tag{5}$$

Relations (5) and (4) provide the equations of motion for the potential part[2] of the system's Lagrangian density. As mentioned in [2], it is generally the case that when (5) or (4) is satisfied, the matter-field current associated with the Lagrangian acts as the source for the gauge fields. This is a consequence of the fact that the matter-field dependent and the gauge-field dependent currents are at separate sides in each of the latter two equations.[3]

The covariant dependence on the velocity-space gauge field is obvious from (4), and it was shown in a similar way for (5) in [2]. The derived conserved currents make a correspondence between the matter fields and the kinetic (velocity-dependent) gauge fields. They open the way to conclude invariance between the sources of the scalar fields on the one side, and the gauge vector fields on the other.

There is easy to see that $F_\alpha^{(1)\mu\nu}(\dot{x})$ and $F_\alpha^{(2)\mu\nu}(x)$ transform in the same way, as isovectors, under a local transformation $V(\dot{x}) \in G$ [1]: $F_\alpha^{(1)'\mu\nu}(\dot{x}) = V^{-1} F_\alpha^{(1)\mu\nu}(\dot{x}) V$ and $F_\alpha^{(2)'\mu\nu}(x) = V^{-1} F_\alpha^{(2)\mu\nu}(x) V$. Notice, that the forms of $J_\alpha^{(1)\nu}(\dot{x})$ conserved currents in the presence of velocity depending fields coincide with the form of those currents that we had obtained for space-time depending fields. With respect to this identical form, as well as to the variety of the symmetry groups that they may obey, one can replace $\varphi(\dot{x}) \to \varphi(x)$, $D(\dot{x}) \to B(x)$ and $J_\alpha^{(1)}(\dot{x}) \to j_\alpha^{(1)}(x)$, where $B(x)$ are familiar physical gauge fields with symmetries, e.g., $U(1), SU(2)$, [and $SU(2) \times U(1)$], $SU(3)$ or $SO(3,1)$, with the substitution of ℷ by the corresponding coupling constants. $F_\alpha^{(1)\mu\nu}(\dot{x})$ take the same forms and transform in a velocity dependent $\mathbf{D}$ gauge field like the components of a $j^\nu(x)$ current and isovectors $f^{\mu\nu}(x)$ of a

[1] The $F^{(2)\mu\nu}$ fields take the general form $F_\alpha^{(2)\mu\nu}(x) = \dfrac{\partial D_{\dot{\rho},\alpha} \partial_\mu \dot{x}^\rho}{\partial x_\nu} - \dfrac{\partial D_{\dot{\sigma},\alpha} \partial_\nu \dot{x}^\sigma}{\partial x_\mu} - i\gimel C^\gamma_{\alpha\beta} D_{\dot{\rho},\beta} \partial_\mu \dot{x}^\rho D_{\dot{\sigma},\gamma} \partial_\nu \dot{x}^\sigma$.

[2] I.e., which serves as the source for the gauge-fields, and consequently as the source for the characteristic charges of the given fields.

[3] Here the only condition assumed was that the field equations be satisfied. No restriction was imposed on the form of the Lagrangian density except that it be invariant under local gauge transformations as defined in (3).

general matter field $\varphi(x)$ and gauge field $\mathbf{B}$, defined by $f^{\mu\nu} = \partial^\nu B_\mu - \partial^\mu B_\nu - ℶ B_\mu \times B_\nu$ in the four dimensional space-time. (This yields the information, that in a boundary situation, i.e., in the absence of relativistic accelerations, our derivation produces the same result as it was known without the assumption of a velocity dependent gauge field. We got back to the results that were known in the absence of a velocity-dependent gauge field, and that were based on calculations in an only space-time dependent gauge field. So, without employing accelerations, we derived the same conserved currents. This justifies our preliminary assumption, that handling the space-time coordinates as implicit parameters not only provides additional information but it preserves the physical relevance of the theory.)

1.3 First conserved quantity: Conservation of the field charge (ד)

We denoted [1] the sources of the individual physical fields (for example, gravitational, electromagnetic, and so on) by the letter ד (dalet, the fourth letter of the Hebrew alphabet) and we call them field charges (for example, mass, electric charge, etc.) which have two isotopic states. The field charges form four-currents each (at least in the Standard Model). In a general case, the T_γ (which appear in the presented conserved currents) as introduced above, are matrix-representation operators generating the group G, with the mentioned commutation rule $[T_\alpha, T_\beta] = iC^\gamma_{\alpha\beta} T_\gamma$. They can be replaced by concrete operators of the concerned fields, according to their characteristic symmetry groups, like $U(1)$, $SU(2)$, $SU(3)$ or $SO(3,1)$, and their combinations, and ב can be substituted by the concrete coupling constants of the individual physical fields. Thus, in a general case, and with group G of an arbitrarily chosen physical field $\mathbf{B}$, one can write $\varphi(x)$ and ℶ in the equations for the currents $J^{(1)\mu}_\alpha$ and substitute the above equations with:

$$J^{(1)\nu}_\alpha(x) = iℶ \frac{\partial L}{\partial(\partial_\nu \varphi_k)} (T_\alpha)_{kl} \varphi_l(x), \quad J^{(1)\nu}_\alpha(x) = \partial_\mu F^{(1)\mu\nu}_\alpha(x),$$

$$F^{(1)\mu\nu}_\alpha(x) = \frac{\partial L}{\partial(\partial_\mu B_{\nu,\alpha}(x))}, \tag{6}$$

and

$$\hat{\partial}_\mu F^{(1)\mu\nu}_\alpha(x) = \partial_\mu F^{(1)\mu\nu}_\alpha(x) + iℶ C^\gamma_{\alpha\beta} B_{\mu,\beta}(x) \times F^{(1)\mu\nu}_\gamma(x)$$

The operators of the quanta of the given physical field are determined by the generators $\{T_\alpha\}$ of the symmetry group of the respective field. The full conserved field charge currents $J^{(1)\mu}_\alpha$ will provide the conserved quantities of the field $\varphi(x)$, which the gauge field $\mathbf{B}$ interacts with. We called these conserved quantities field charges and denoted by ד. (We will see in subsections 1.4 and 1.5 that ד appears in two isotopic states, what we call isotopi field charges.) We can get the conserved quantity by integration of the current in the usual way, applying Gauss' theorem, where the integral of the spatial components vanishes at an infinite boundary, and we get:

$$\frac{d}{dt} \frac{ℶ}{c} \int \frac{\partial L}{\partial(\partial_4 \varphi_k)} (T_\alpha)_{kl} \varphi_l(x) dV = 0 \tag{7}$$

where the integral provides the conserved field charge ד of the source field φ.

The results derived in this subsection coincide with the well known conservation laws of field theories. We treat it here in order to make it comparable with the results of the next subsection (1.4), and to demonstrate that the two conserved quantities appear simultaneously (Sec. 1.5).

1.4 Second conserved quantity: Conservation of the isotopic field charge spin ($\varDelta$)

What is isotopic field charge spin (IFCS)? [1] assumed that the field charges appearing in the potential part of a Hamiltonian as the scalar sources of a matter field, and the field charges appearing in the kinetic part of a Hamiltonian, and in currents as sources of gauge fields are qualitatively different physical quantities. They are called isotopic field charges. For example, the mass of gravity and the mass of inertia are considered here, and from now on, as two, qualitatively different physical properties (although equal in their values in rest), and serve as the sources of gravitational and kinetic fields, respectively. In a similar way, the electric charges appearing in the Coulomb potential and the electric charges appearing in the currents that serve as sources of magnetic fields are also qualitatively different physical quantities. The same is assumed on the sources of other interaction fields. These twin couples of physical quantities, like isotopes of each other, are called with the common name isotopic field charges.

This means, there appear two different isotopes of a given field charge in the individual elements of a four-current. This distinction between the isotopic field charges would distort the Lorentz invariance of these currents, what is not in accordance with our physical experience. Therefore, the assumption of the distinction between the isotopic field charges must involve the assumption that they are members of a group whose elements can be transformed into each other. This symmetry among the members of an individual isotopic field charge couple counteracts the symmetry lost by the introduction of the distinction between them. This new invariance can be represented by an $SU(2)$ group, which rotates the two isotopic states of the field charges in a gauge field, and can take two stable positions. [1] (a) proved (as cited below) that the introduced velocity dependent $\mathbf{D}$ gauge field serves as the field where the isotopic states of the field charges are rotated, (b) introduced that the rotated property (the two stable states of the isotopic field charges) be called (by analogy) isotopic field charge spin ($\varDelta$), and (c) proved that the conservation of the $\mathbf{J}^{(2)}$ currents provides the conservation of the isotopic field charge spin. So, the above cited, derived $J_\alpha^{(2)\nu}(x)$ are the isotopic field charge spin currents, which — similar to $J_\alpha^{(1)\mu}$ — are also conserved and yield a conservation law. The conserved quantity derived from $J_\alpha^{(2)\nu}(x)$ is the isotopic field charge spin $\varDelta$.

The conserved current in the kinetic field can be read from (3). The right side of (3) represents the full conserved isotopic field charge spin current, which includes the contribution of the $\mathbf{D}$ field.[4]

We have introduced the $\mathbf{D}$ field — which is shown to be responsible for the isotopic field charge spin transformation — to counteract the dependence of a $V(\dot{x}) = e^{-ip_\alpha(\dot{x})T_\alpha}$ transformation on $\dot{x}_\mu$. The field equations, which are satisfied by the twelve independent components of the $\mathbf{D}$ field, and their interaction with any field that carries isotopic field charge spin are unambiguously determined by the defined currents and covariant $F^{(2)\mu\nu}$-s constructed from the components of $\mathbf{D}$. Considering a general Lorentz- and gauge invariant Lagrangian, we obtain from the equations of motion that $J^{(2)1,2,3}$ and $J^{(2)4}$ are, respectively, the isotopic field charge spin current density and isotopic field charge spin ($\varDelta$) density of the system. The total isotopic field charge spin

$$\Delta = \frac{i}{\gimel} \int J^{(2)4} \mathrm{d}^3 x \tag{8}$$

is independent of time and independent of Lorentz transformation. $J^{(2)\mu}$ does not transform as a vector, while $\varDelta$ transforms as a vector under rotations in the isotopic field charge spin field.

[4]Similar attempts (like our in the velocity space) were made by [6] in the phase space (with a particular mapping from the configuration space to phase space), and they anticipated the quantization of the models.

1.5 Coupling of the two conserved quantities ($\urcorner$ and $\varDelta$)

The dependence of the two currents $J_{\alpha}^{(1)\mu}$ and $J_{\alpha}^{(2)\mu}$ on each other has physical consequences. Once, it justifies that the quantities, whose conservation they represent and which are coupled, exist simultaneously. Secondly, the coupling of a conserved quantity in a space-time dependent field — which coincides with one of our familiar physical fields — with another in a kinetic (velocity dependent, introduced in [2] and [1]) gauge field indicates that *the derived conservation verifies* just *the invariance between the two isotopic states of the field charges, namely between the potential* $\urcorner_V$ *and the kinetic* $\urcorner_T$ (where the indices V and T refer to the potential and the kinetic components of a Hamiltonian, respectively, and the two kinds of $\urcorner$ correspond to the two isotopic field charges). (Remember that $\urcorner$ can be field charges of different physical fields marked in common with $\mathbf{B}$, while $\varDelta$ represents a single quantity belonging to the kinetic gauge field $\mathbf{D}$).

In the presence of kinetic fields we have two conserved currents that are effective simultaneously. The kinetic gauge field $\mathbf{D}$ is present simultaneously with the interacting matter $[\varphi]$ and gauge $[\mathbf{B}]$ fields. The presence of $\mathbf{D}$ corresponds to the property of the field charges $\urcorner$ of the individual fields that they split in two isotopic states, and analogously to the isotopic spin, we named these two states *isotopic field charge spin* (IFCS) what we denote by $\varDelta$. The source of the isotopic field charge spin ($\varDelta$) is the field $\varphi(\dot{x})$ in interaction with the kinetic gauge field $\mathbf{D}$.

In summary, the physical meaning of $\varDelta$ can be understood, when we specify the transformation group associated with the $\mathbf{D}$ field, which describes the transformations of $\urcorner$ (i.e., the isotopic field charges). $\urcorner$ can take two (potential and kinetic) isotopic states $\urcorner_V$ and $\urcorner_T$ in a simple unitary abstract space. Their symmetry group is $SU(2)$, that can be represented by 2×2 T_α matrices. There are three independent T_α that may transform into each other, following the rule $[T_\alpha, T_\beta] = iC^{\gamma}_{\alpha\beta}T_\gamma$, where the structure constants can take the values 0, ± 1. Let T_1 and T_2 be those which do not commute with T_3; they generate transformations that mix the different values of T_3, while this "third" component's eigenvalues represent the members of a $\varDelta$ doublet. For the isotopic field charges compose a $\urcorner$ doublet of $\urcorner_V$ and $\urcorner_T$, the field's wave function can be written as

$$\psi = \begin{pmatrix} \psi_T \\ \psi_V \end{pmatrix}. \tag{9}$$

(9) is the wave function for a single particle which may be in the "potential state", with amplitude ψ_V, or in the "kinetic state", with amplitude ψ_T. ψ in (9) represents a mixture of the potential and kinetic states of the $\urcorner$, and there are T_α that govern the mixing of the components ψ_V and ψ_T in the transformation. T_α ($\alpha = 1, 2, 3$) are representations of operators which can be taken as the three components of the isotopic field charge spin, $\varDelta_1$, $\varDelta_2$, $\varDelta_3$ that follow the same (non-Abelian) commutation rules as do the T_α matrices, $[\varDelta_1, \varDelta_2] = i\varDelta_3$, etc. These operators represent the charges of the isotopic field charge spin space, and ψ are the fields on which the operators of the gauge fields act.

The quanta of the $\mathbf{D}$ field should carry isotopic field charge spin $\varDelta$. The $\varDelta$ doublet, as a conserved quantity, is related to the two isotopic states of field charges ($\urcorner$), and the associated operators ($\varDelta_{\mathrm{i}}$) induce transitions from one member of the doublet to the other.

1.6 Interpretation of the isotopic field charge spin conservation

Invariance between $\urcorner_V$ and $\urcorner_T$ means that they can substitute for each other arbitrarily in the interaction between field charges of any given fundamental physical interaction. They appear at a probability between $[0, 1]$ in a mixture of states in the wave function

$$\psi = \begin{pmatrix} \psi_T \\ \psi_V \end{pmatrix}$$

so that the Hamiltonian of a *single particle* oscillates between V and T, while the Hamiltonian of a *composite system* is a mixture of the oscillating components of the particles that constitute the system. The individual particles in a *two-particle system* are either in the V or in the T state respectively, and switch between the two roles permanently; while the observable value of H is the expected value of the mixture of the actual states of the two, always opposite state particles. In the case of mechanics this means that the mass of any physical object is a mixture of unit masses of gravity and unit masses of inertia that oscillate between the two states each. In gravitational interaction between two unit masses, one of them is in gravitational state, and the other in kinetic state. They swap their roles permanently by the exchange of the quantum of the Δ field.

The invariance between $\daleth_V$ and $\daleth_T$ (what is ensured by the conservation of Δ), and their abilitiy to swap means also that they can restore the symmetry in the physical equations which was lost when we replaced the general $\daleth$ (namely mass m, electric charge $q, \ldots$ etc.,) by their isotopes $\daleth_V$ and $\daleth_T$.[5]

2 Finsler geometry in the presence of isotopic field charges

Let us specify (5) for the gravitational field [9]. The right side of the equation contains the scalar field that serves for the source of the gravitational field. The $\gimel$ can be replaced by the gravitational coupling constant g. As we noticed, the dependence on the gauge fields is on the left side of the equation (5). $F_\alpha^{(1)\mu\nu}(\dot{x})$ must satisfy the

$$T_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} + \frac{1}{4}\delta_{\mu\nu}g^{k\sigma}F_{\lambda\sigma}g^{\lambda\rho}F_{k\rho}$$

identity for the energy-momentum tensor $T_{\mu\nu}$. (In order to bring this form in compliance with the indices in (5), one should raise the indices by multiplying with the metric tensor $g_{\beta\gamma}$ in the right side.) This energy-momentum tensor $T_{\mu\nu}$ can be expressed by the way of the Einstein equation

$$T_{\mu\nu} = -\frac{1}{8\pi G_N}(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}) \tag{10}$$

where $R_{\mu\nu}$ is the Ricci tensor defined by the help of the derivatives of the metric tensor $g_{\mu\nu}$, R is the Ricci scalar formed from the Ricci tensor (Riemann curvature) and the metric tensor, and Λ is a constant of Nature, as well as G_N the constant of Newton.

The metric tensor $g_{\mu\nu}$ and its derivatives depend on the localisation of the given point in the space-time in the General Theory of Relativity (GTR), and are subject of Riemann geometry. In the presence of a kinetic field, that means, isotopic mass field $\mathbf{D}$ (mass being the field-charge of the gravitational field), however, the curvature depends also on velocity. (Whose velocity? On the actual inertial velocity of a test unit-mass placed in a given space-time point in the reference frame fixed to the source of a scalar gravitational field φ which appears on the right side of (5).) The $g_{\mu\nu}$ metric tensor, and consequently the affine connection field and the curvature tensor formed from its derivatives, depend on space-time and velocity co-ordinates. With the appearance of the dependence on the velocity vector, the curvature becomes dependent on its direction in each space-time point. The direction (additional parameter) attributed to each space-time point is defined by the orientation of the velocity of a test unit-mass in the given space-time point, $\frac{\mathbf{v}}{|v|}$. The curvature can no more follow a "simple" Riemann geometry, it follows a Finsler geometry whose metric is defined by the dependence of $g_{\mu\nu}$ on (x_σ and) $\dot{x}_\rho$.

[5] Consequences of the application of effective field theories were analysed e.g., in philosophy by [7] and in physics by [8].

Of course, the space-time plus four-velocity dependence of the metric tensor $g_{\mu\nu}$ affects its all derivatives, including the formation of the affine connection field (from first derivatives) and the Riemann curvature (or Ricci tensor, second, covariant derivative)

$$\Gamma_{\lambda\mu\nu} = \frac{1}{2}\left[\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu}\right] \qquad \Gamma^\lambda_{\mu\nu} = g^{\lambda\rho}\Gamma_{\rho\mu\nu}$$

and

$$R_{\mu\nu} = \partial_\mu \Gamma^\lambda_{\nu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\lambda} - \Gamma^\lambda_{\sigma\lambda}\Gamma^\sigma_{\mu\nu}.$$

The solution of the Einstein equation in velocity dependent field with Finsler geometry must necessarily lead to solutions different from that of Schwarzschild.

3 The role of the isotopic field charge spin conservation

The role of equation (4) is to retain the invariance between the two isotopic forms, namely gravitational and inertial, of masses. The importance of this is to save the covariance of our equations. Since there appear two different kinds of (isotopic) masses in the energy-momentum "four-vector" (in the fourth column of $T_{\mu\nu}$,) it does no more transform as a vector, and Lorentz transformation can no more guarantee alone the covariance of our equations.

As a consequence of the distinction between m_V and m_T, as well as the association of the energy content with the mass m_V and the components of the momentum with m_T, we lose also the symmetry of the $T_{\mu\nu}$ energy-momentum tensor. To retain symmetry in Einstein's field equations we must require again the invariant transformation of m_V and m_T into each other in an appropriate gauge field, namely in **D**. We refer to [10] who foresaw the possible generalisation of YM type gauge invariance in general relativity "in close analogy with the curvature tensor". If we consider the energy-momentum tensor (in which both isotopic states of mass appear) as the source of the gravitational field, then — in the usual way — the scalar and the vector potential can be separated. See, m_4 in T_{44} does not compose a fourth component of a four-vector in the classical theory of gravitation where there is a single scalar mass. If we consider now $m_4 = m_V$, the three components of the kinetic mass m_T can compose a three-vector, however $T_{\mu 4}$ will not form a four vector either.

To maintain the Lorentz invariance of our physical equations in the gravitational field, we must demand to restore the invariance of $\begin{pmatrix} \vec{m}_T \\ m_V \end{pmatrix}$ under an *additional transformation* that should *counteract the loss of symmetry caused by the introduction of two isotopic states of mass.* We discussed that transformation in section 1. Further, in the case of gravitation the relation of the scalar and the vector fields are not linear even if we have not made distinction between the potential and kinetic masses. The non-linearity is coded in the relation of the tensors [11] at the right side of the Einstein equation (10) (in units $c = 1$), or we can write $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$ where the Einstein tensor is defined as $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ whose covariant derivative must vanish.

Since our $T_{\mu\nu}$ tensor has already lost its symmetry, we can take $\Lambda g_{\mu\nu}$ into account within a modified $T'_{\mu\nu}$ — handling the gravitational and kinetic masses in it together with the dark energy — and we get the following formally symmetric equation: $G_{\mu\nu} = 8\pi G T'_{\mu\nu}$.

The symmetry of the energy-momentum tensor can be saved by the invariant gauge transformation of the IFCS. The most important analogy is between the behaviour of the potential and the kinetic field charges of the individual fields that makes probable to postulate a unique transformation to assure their invariance (cf., section 1).[6] So the invariance under the Lorentz

[6] As [12] stated, "In contrast to the symmetry or *invariance* requirement in STR, the principle in GTR is

transformation combined with the invariance of the isotopic field charge spin field provide together the covariance of the gravitational equation. However, this combined transformation should now be taken into consideration in a field with a metric depending on all space-time and velocity co-ordinates, following a Finsler geometry.

Appendix

Comparison of the invariance properties in classical GTR and in the IFCS model.

In classical physics, conservation laws — as consequences of the invariance properties of the investigated systems — can be obtained by integration of the Euler-Lagrange equations of motion of classical mechanical point systems. According to Hamilton's principle the variation of the action integral of the system's Lagrangian must be zero. These conservation laws include the conservation of the energy — invariance under translation in time. That conserved energy is equivalent with a well determined amount of mass $E = mc^2$, where $m = m_V$ is gravitational mass, and this conservation law does not provide any information on the quantity of kinetic mass.

In general relativistic treatment, the source of the gravitational field is the $T_{\mu\nu}$ momentum-energy stress tensor, which includes the sources of inertial and gravitational effects as well. Applying the same variational method and integration for the Einstein equation (using $[+ + + -]$ signature) we derive the conservation of the $-T_{44}$ element of the $T_{\mu\nu}$ momentum-energy stress tensor. $-T_{44}$ is energy density of the gravitational field, and is proportional to a certain amount of mass. According to invariance under translations in the Minkowski space (Lorentz transformation) the conserved current can be written in the form

$$\partial_\mu T_{\mu\nu} \equiv \partial_\mu \left(L\delta_{\mu\nu} - \partial_\nu \varphi_r \frac{\partial L}{\partial \partial_\mu \varphi_r} \right) = 0$$

where φ_r denote functions on which (and their first derivatives) the Lagrangian may depend.

The Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

provides the elements of $T_{\mu\nu}$ in which — according to the left side — the contribution of the kinetic and potential components are mixed by the $g_{\mu\nu}$ curvature tensor. Applying the usual integration method and Gauss' theorem, we get the fourth column of the momentum-energy stress tensor for a conserved quantity, what is no else than the four-momentum density, which behaves like a four-vector and whose individual components are

$$P_\nu = \frac{1}{ic} \int T_{4\nu} dV$$

or separated

$$P_k = \frac{1}{ic} \int T_{4k} dV = \frac{1}{ic} \int \partial_k \varphi_i \frac{\partial L}{\partial \partial_4 \varphi_i} dV \qquad (k = 1, 2, 3);$$

most often presented as strictly speaking a *covariance* requirement." Gauge theories behave like GTR, at least in this respect. General covariance "is not tied to any geometrical regularity of the underlying spacetime, but rather the form invariance (covariance) of laws under arbitrary smooth coordinate transformations" [12, p. 34]. [13] found that the more general geometry resulting from admitting local changes called gauges described not only gravity but also electromagnetism. He showed also that the conservation laws of Noether follow in two distinct ways in theories with local symmetries. This led to the Bianchi identities, which hold between the coupled equations of motion, and which are due to the local gauge invariance of action. Later [14] demonstrated that the conservation of the electric charge followed from the local gauge invariance in the same way as does energy-momentum conservation from co-ordinate invariance in GTR.

$$H = icP_4 = -\int T_{44} dV = \int \left(\partial_4 \varphi_i \frac{\partial L}{\partial \partial_4 \varphi_i} - L \right) dV$$

what are considered the conserved total momentum and energy of the field respectively.

If we take into account the qualitative difference between the masses m_T (what appear in the components of P_k) and m_V (what appears in H) that are mixed by the curvature tensor $g_{\mu\nu}$ in the elements of $T_{\mu\nu}$, this consideration will involve the mixed m_T and m_V dependence of the Lagrangians as well. As a consequence, P_k and H cannot be considered separately, and independently of each other, conserved quantities. (We do not investigate here the ambiguous interpretations of invariant mass.) The covariance of the gravitational equation can no more be secured by the Lorentz invariance alone. The lost symmetry of nature can be restored only with the shown invariance between the isotopic mass states (as field charges of the gravitational field, conservation of Δ) which are rotated in an isotopic field charge spin gauge field. The covariance of the gravitational equation is a result of invariance under the combination of the Lorentz transformation and rotation in the isotopic field charge field. In the latter case the four components of ($P_k[m_T]$, $H[m_V]$) transform as isovectors. Due to the IFCS gauge transformation, the transformation of the field components can be described in a (space-time +) velocity dependent gauge field, whose metric, consequently, depends also on the velocity components, and is subject of a Finsler geometry.

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ФИНСЛЕРОВА ГЕОМЕТРИЯ ОТО В ПРИСУТСТВИИ ЗАВИЯСЯЩИХ ОТ СКОРОСТИ КАЛИБРОВОЧНЫХ ПОЛЕЙ

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Поля в окрестности источников задают искривленную геометрию. Зависящие от скорости явления в этих полях требуют для своего описания тензор кривизны, элементы которого зависят от величины и направления скорости движения источника взаимодействующих калибровочных полей в системе отсчета поля материи. Эта двойная зависимость кривизны от пространства-времени и скорости требует для своего описания финслерову геометрию.

Ключевые слова: фундаментальные взаимодействия, заряды поля, инвариантность, ковариантность, изотопический спин зарядов поля, сохранение, симметрия, калибровочный бозон, поля зависящие от скорости, финслеров тензор кривизны, модифицированное уравнение Эйнштейна.