FOUR-DIMENSIONAL TIME

D. G. Pavlov

Moscow State Technical University n. a. N. E. Bauman
hypercomplex@mail.ru

The generalized metric space, that can be called the flat four-dimensional time, is based on the Berwald-Moore's Finslerianview of metric function. This variety let us introduce physical notions: the event, the world lines, the reference frames, the multitude of relatively simultaneous events, the proper time, the three-dimensional distance, the speed, etc. It is demonstrated how from the point of the physical observer, associated with the world line, in absolutely symmetrical four-dimensional time the contraposition of the coordinate takes place, that defines its proper time, with the ones that appear as the result of the measurements made with the help of sample signals. When the signals correspond with lines, which are practically parallel to the world line of the observer, he starts to see the three-dimensional space which at the limit is the Euclidean space.

1. Introduction

For the last 100 years the idea, that the Pseudo-Euclidean metric with an alternating-sign quadratic dependence on the length of the vectors from the magnitude of its components lays in the basis of geometry, has taken root in physics. But still numerous and various attempts to connect all the known natural forces nature with the metric and make true the idea of the total geometrization of physics have failed. This drives to the idea that the reason lies not in the lack of scientists' creativity, but in the metrics itself, even better to say in the classical quadratic form, in place of which it is admittedly to use other dependences. Unfortunately, this attitude, the possibility of which indicated Riemann [1], was for the first time studied by Finsler [2], and up to nowadays used by hundreds of investigator [3], did not give eventual pictures. Though nowadays the work in this direction is continued, it considerably differs from many of them, as it is based on the idea of scalar poly-products, which is new for the Finslerian geometry, and metric form that is connected with one of the most fundamental notions in mathematics – the real number.

2. Multidimensional time .

The spaces that have unique correspondence with algebras, that are the sum of several real number algebras, stand out from Finslerian linear spaces. The metric functions do not depend on the point and in one of the bases look like:

$$F(x') = \left|\prod_{i=1}^{n} x'_i\right|^{1/n}, \tag{1}$$

where x'_i are the components of the vector and n is the number of dimensions. Such metric functions are well-known in the theory of Finslerian spaces and took the name of Berwald-Moore's function [3].

Geometries with such metrics in many ways are of the same type and the difference is related only to the dimension. The total equality of all non-isotropic directions is their main peculiarity. As any of such directions can be related to the proper time of the inertial reference frame, it is appropriate to call such spaces the *multi-dimensional time.*

Note. It seems that it is possible to relate a general line with an inertial reference frame in any linear space, where the element of the length is defined in every point. But in many spaces some reference frames do not admit the presence of isotropic connections with other lines that go in a parallel way with the given. For the viewer related to such reference frames, the existence of isotropic vectors, with which it is traditional to associate the light signals, becomes the origin to the idea of the physical distance and consequently the physical space.

The defined in this way spaces not always have the same shape as the one we got used to (in every day life and thanks to Euclid and Minkowski). At the same time we have to put a more general meaning than usually into the idea of physical space. On the other hand nothing prevent us from considering that in the sectors or dimensions, where isotropic connection is not set or have an extraordinary characteristics, that physical directions are undetectable, though representable from geometrical point of view. Consequently, it is quite logical to suppose the existence of some spaces, some parts of directions and even dimensions of which are not apparent from their physical side. From such point of view it would be interesting to analyze arbitrary linear spaces and in particular those, connected with quadratic forms and the Berwald-Moore's metrics treated over the field of complex numbers.

The chosen geometrical element of every n-dimensional time is its isotropic sub-space, that is a figure constructed from n-hyperplanes, that divide the multiformity into 2^n-equal simply connected cameras. Any of the cameras adjoins to the others, but for the facing, with which it borders in a point. The adjoining cameras can be classified according to the distinguished by the dimension of the frontier planes from 1 to $(n-1)$. All simply connected cameras are equal and have the shape of regular pyramids, n-hyperplanes of which start from the top and go to the infinity. We will call such pyramids, by analogy with isotropic cones of the Minkowski space, the light pyramids. Every *light pyramid* has n one-dimensional edges that can easily be connected with a special basis. In the basis the geometrical correlation of the multy-dimensional time appears in a vivid shape and, as such a basis is to permutation unique, it is quite natural to call it the *absolute*.

Any single vector that belongs to the inner area of a light pyramid can be continuously introduced into any other single vector that belongs to the same pyramid. The respective transformation form $n-1$-parametrical Abelian subgroup of movements, that leaves the initial metric function (1) invariant. The metrics of such transformations in the absolute basis is reduced to the diagonal shape:

$$\begin{pmatrix} a'_1 & 0 & \dots & 0 \\ 0 & a'_2 & \dots & 0 \\ . & . & \dots & . \\ 0 & 0 & \dots & a'_n \end{pmatrix}, \tag{2}$$

where $\prod_{i=1}^{n} a'_i = 1$. The corresponding reflections can be classified as Hyperbolic turn (that in a way are analogous to the busts of the pseudo-Euclidean spaces) because such transformations leave on the place a point of convergence of the tops of all the pyramids and isotropic edges of the last at the same time turn into themselves. Among continuous movements of the multy-dimensional time along with hyperboloic turns there is also a n-parametrical subgroup of parallel transfers. The examined variety doesn't include any other continuous congruent transformations and that is why has less freedom than the spaces with quadratic types of metrics. The very circumstance made Helmholtz, Lee, Weyl prove a number of theorems that stated that the oneness of the quadratic metrics [4−6]. The main emphasis was made to maximum mobility in quadratic spaces. This according to them gave grounds to reject all other metric forms in the meaning of the basis of the real space-time. Let us note without rejecting the theorem accuracy that its approval is based on the examination of only the distinguished linear transformations, which means that it gives a chance to other theorems, where non-linear symmetries play the same role. In contrast to continuous congruent transformations the discrete group of symmetry of the multy-dimensional time excels the corresponding Euclidean- and pseudo-Euclidean spaces, but this is not enough to compete with the latter one. What really makes the multy-dimensional time the multy-dimensional time interesting is the presence of distinguished groups of non-linear transformations which are practically as fundamental as the groups of movements.

Such transformations save invariant not the intervals, but specific scalar forms of several vectors, that do not have direct analogous quadratic spaces, and that is why are not well-studied.

It is better to come to the understanding of such polyforms through the generalizing of the idea of the scalar product. It turns out that in a number of Finslerian linear spaces the poly-linear symmetry form of n vectors [7] (its special case is the classical bilinear form) can play the role of the scalar product. Let us call the poly-linear form the *scalar poly-product*. Founding on this generalizing we can enlarge with some Finslerian spaces such fundamental ideas of geometry as the length, the angle, the orthogonality, etc., the introduction of which is difficult due to some problems [8].

In the absolute basis the *scalar poly-product* of the multy-dimensional time looks like:

$$(\mathbf{A}, \mathbf{B}, \ldots, \mathbf{Z}) = \frac{1}{n!} \sum_{(i_1, i_2, \ldots, i_n)} a'_{i_1} b'_{i_2} \ldots z'_{i_n}, \quad \text{at} \quad i_j \neq i_k, \quad \text{if} \quad j \neq k. \tag{3}$$

It is not difficult to believe that with $\mathbf{A} = \mathbf{B} = \ldots = \mathbf{Z}$ the form (3) turns into the metric function (1). We can build the geometry of the linear time in an arbitrary natural scale using the poly-linear symmetrical form (3). But let us focus on this case if we base on common ideas about physical measurements and vivid typological detailedness of the four-dimensional space [9].

3. Four-dimensional time

According to (3) the scalar poly-product, that defines the four-dimensional time, in the absolute basis looks like:

$$(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}) = \frac{1}{4!} \sum_{(i_1, i_2, i_3, i_4)} a'_{i_1} b'_{i_2} c'_{i_3} d'_{i_4}, \qquad \text{when} \quad i_j \neq i_k \text{ if } j \neq k, \tag{4}$$

it follows that the fourth degree of the vector length of such linear space is defined by the expression:

$$(\mathbf{X}, \mathbf{X}, \mathbf{X}, \mathbf{X}) = |\mathbf{X}|^4 = x'_1 x'_2 x'_3 x'_4. \tag{5}$$

While turning to the basis analogous to the orthonormalized [7] (it is more visual than in the absolute case) the expression transforms into a more complicated but still symmetrical form:

$$|\mathbf{X}|^4 = x_1^4 + x_2^4 + x_3^4 + x_4^4 - 2(x_1^2 x_2^2 + x_1^2 x_3^2 + x_1^2 x_4^2 + x_2^2 x_3^2 + x_2^2 x_4^2 + x_3^2 x_4^2) + 8 x_1 x_2 x_3 x_4. \tag{6}$$

In a number of cases it is more convenient to use the form picking out one of the coordinates, in particular x_1:

$$|\mathbf{X}|^4 = x_1^4 - 2(x_2^2 + x_3^2 + x_4^2)x_1^2 + 8(x_2 x_3 x_4)x_1 + (x_2^4 + x_3^4 + x_4^4 - 2x_2^2x_3^2 - 2x_2^2x_4^2 - 2x_3^2x_4^2). \quad (7)$$

The main arguments in favor of the chance of confronting the four-dimensional time to the real physical world is the presence of a group of continuous symmetries [10], that can be examined as an alternative to the linear group of spatial turning of the Minkowsky space. Not a scalar poly-product of the four-dimensional time (4) is an invariant to the transformations, but a specific form, that is defined by 2 vectors:

$$S(\mathbf{A},\mathbf{B}) = \frac{(\mathbf{A},\mathbf{A},\mathbf{A},\mathbf{B})}{(\mathbf{A},\mathbf{A},\mathbf{A},\mathbf{A})^{1/2}} + \frac{(\mathbf{A},\mathbf{B},\mathbf{B},\mathbf{B})}{(\mathbf{B},\mathbf{B},\mathbf{B},\mathbf{B})^{1/2}}. \quad (8)$$

Though the form $S(\mathbf{A},\mathbf{B})$ is not an additive quantity of the vectors that belong to the interior of domain of a light pyramid, it complies with other very important characteristics of the common scalar product, to be more specific: the symmetry, the rule of multiplication by the vector, the sign distinctness and the triangle rule [10]. According to this there exists a principal opportunity in the four-dimensional time to introduce the idea of the three-dimensional distance, that corresponds to most of common conceptions of the physical quantity, but for the additivity. From philosophical point of view the last characteristic is very important. No, really, why should the rule of composition differ from the one of three-dimensional distances, as both values are relative? Such linearity appears only when we work with big distances, as well as the non-linearity of the rule of speed composing is essential only in the relativist field. At the same time an additional fundamental constant – the maximum possible magnitude of the physical system, or, in other words, the radius of the Universe, acts as the light speed in the three dimensional distance. For everyday distances we can still use the linear approximation, but in the space scale, in case of logical appliance of the multy-dimensional time conception, certain corrections should be made.

4. Plenty of relatively simultaneous events

We should first of all clarify the situation about a number of simultaneous events to give the definition of the four-dimensional time, three-dimensional speed and distance. Let us understand under it the total of points equidistant (of course in the meaning of the accepted Finslerian metrics (5)) from a pair of fixed events. In contrast to the Minkowskian space, where a multitude of points constitute hyperplanes, in the four-dimensional time the corresponding planes are non-linear [10]. Their form depends not only on the direction of the world line, that connects the fixed points, but also on the magnitude of the interval that separates them. This is the most fundamental difference from the space of the Special Theory of Relativity, as the idea of simultaneosity is defined now not only by the speed of the reference frame, but also by the interval of time that separates the instantaneous position of the observer and the examined spatial layer of events. So the relativism in the four-dimensional time touches upon not only the hyperbolic turns, with the help of which realizes the switch between one system to another, but also the transmission, that enables to change the reference point.

From philosophical point of view such generalization is quite logical, but in fact establishes a sort of relationship between the two subgroups of the total group of congruent symmetries. As an indirect affirmation of the made conclusion can serve the fact that in algebra transmissions lack the operation of composition, which are a part of the four-dimensional time, and hyperboloic turnings - multiplication, and mathematics do not question relationship between them. A natural way of introducing the idea of the physical distance in the four-dimensional time is offering a method that from conceptual point of view is analogous to the method of defining of the idea in the Minkowskian space. By definition under distance we can understand a value that equals (or is proportional) the tie intervals, that go along the world line of the observer, between sending some uniformly moving model signals to the world lines of the examined objects, and receiving the reflected signals. It leads to the fact that it is senseless to use the idea of distance towards single events in the four-dimensional time, and is productive concerning only chains of them, that are presented by certain lines. We can pay no attention to the fact in the Minkowskian space, as multitudes regarding simultaneous events are hyperplanes, as a result the distance defined for an arbitrary pair of parallel lines were still substantial and for a pair of points.

Not to overload the brief article with excessive community, but at the same time to be rather specific, we will give the result to which the described above algorithm drives only in one case - when the world line of the observer coincides with the real axis, it itself is situated at the point $(T, 0, 0, 0)$ and the necessary layer goes through the point $(0, 0, 0, 0)$ (Fig. 1) [Here and later on the appearing coordinates relate to the generalized orthogonal basis [7] that differs tremendously from the absolute].

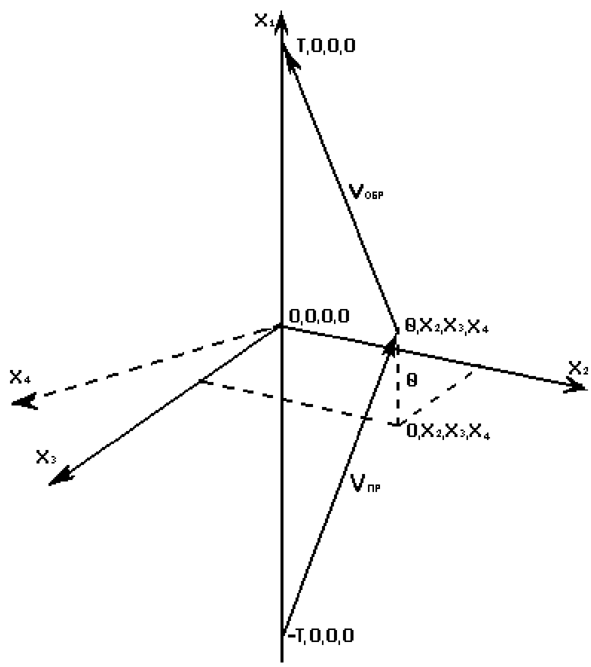

Figure 1: The world lines of direct and opposite signals with speed module

In this case the equalization, that relates the real coordinate θ of a point of the plane simultaneity to three other coordinates x_2, x_3 and x_4, follows from the rule of equality of the vector length that have the following components $(T+\theta, x_2, x_3, x_4)$ and $(T-\theta, -x_2, -x_3, -x_4)$. (Variable θ means deviation of concrete point from hyperplane $x_1 = 0$.) Using the expression for the magnitude of the interval (7) and at the same time concerning that for even degrees $(-x)^n = x^n$, we have:

$$(T+\theta)^4 - 2(x_2^2 - x_3^2 + x_4^2)(T+\theta)^2 + 8(x_2x_3x_4)(T+\theta) + (x_2^4 + x_3^4 + x_4^4 - 2x_2^2x_3^2 - 2x_2^2x_4^2 - 2x_3^2x_4^2) =$$
$$(T-\theta)^4 - 2(x_2^2 + x_3^2 + x_4^2)(T-\theta)^2 + 8(x_2x_3x_4)(T-\theta) + (x_2^4 + x_3^4 + x_4^4 - 2x_2^2x_3^2 - 2x_2^2x_4^2 - 2x_3^2x_4^2).$$

Opening the brackets and collecting terms we get:

$$T\theta^3 + (T^2 - x_2^2 - x_3^2 - x_4^2)T\theta + 2x_2x_3x_4T = 0. \quad (9)$$

introducing sizeless value $\eta = \theta/T, \chi_2 = x_2/T, \chi_3 = x_3/T, \chi_4 = x_4/T$ and taking into consideration that $T \neq 0$ we get a cubic equalization relatively to η:

$$\eta^3 + (1 - \chi_2^2 - \chi_3^2 - \chi_4^2)\eta + 2\chi_2\chi_3\chi_4 = 0. \quad (10)$$

Its real root characterizes the relative value of deflection of the simultaneity plane absciss from the coming through its center according to the hyperplane $x_1 = 0$. We will call such parameter the *coefficient of non-platitude.* When $\chi_2 \approx \chi_3 \approx \chi_4 \to 0$, η also stems to 0, we mean around the point $(0, 0, 0, 0)$ the plane of the simultaneity turns into the hyperplane $x_1 = 0$.

The plane of simultaneity has physical meaning only inside the light pyramide, that has the world line of the observer, in other case it would be necessary to admit the physical meaning of the superlight speed. Following the method of the Special Theory of Relativity, with every vector that start at $(-T, 0, 0, 0)$ and ends at the plane of simultaneity, or in other words at $(\eta T, x_2, x_3, x_4)$ it would be quite natural to connect the world line of the signal, that has a definite uniform speed. We will transform the signals of the vectors, if they have equal interval values, according to the value of the speed module: $|\mathbf{V}_{\text{dir}}|$. Logically the signal, that is confronted to the vector, connecting the points $(\eta T, x_2, x_3, x_4)$ and $(T, 0, 0, 0)$, has the value that is inverse to the speed $|\mathbf{V}_{\text{rev}}|$. On contrast to the Minkowskian space such vectors have components that differ not only in sign but also in value (Fig. 1), to be more specific: $\mathbf{V}_{\text{dir}} \leftrightarrow (\eta T + T, x_2, x_3, x_4)$ and $\mathbf{V}_{\text{rev}} \leftrightarrow (T - \eta T, -x_2, -x_3, -x_4)$. In the Minkowskian space the coefficient of the non-platitude η for every point of the plane of the simultaneity equals 0, as the result the components of the vectors that correspond to direct and inverse signal look like: $\mathbf{V}_{\text{dir}} \leftrightarrow (T, x_2, x_3, x_4)$ and $\mathbf{V}_{\text{rev}} \leftrightarrow (T, -x_2, -x_3, -x_4)$.

To give a definition of distance between the real axis and an arbitrary line parallel to it, which is totally defined by 3 fixed coordinates x_2, x_3, x_4, we should have a model signal, or even better to say vectors related to it, with the help of which it is possible to make intervals that would equal the distance of different directions. As well as in the space of the Special Theory of Relativity, in the four-dimensional time it is more convenient to relate such symbol signals to isotropic vectors, that at one end have the same beginning and from the other - they set against the plane of simultaneity. In the Minkowskian geometry a number of ends of such vectors represent an intersection of two light cones: the future with the top at point $(-T, 0, 0, 0)$ and the past whose top is deposed to $(T, 0, 0, 0)$. As is well known the result of such interception is a common sphere, that lies completely in the hyperplane $x_1 = 0$. This is typical only for spaces with a quadratic metric type. In any case in the fur-dimensional time an analogous figure that is the result of interception of two facing light pyramids, is not plane though consists of linear elements.

Tit is better to make sure of it using the three- and four-dimensional time [12] as the example, in particular looking at Fig. 2 where it is demonstrated the interception of two light pyramids. For comparison, an interception of two light cones of the three-dimensional pseudo-Euclidean space is demonstrated on the same picture. In the three-dimensional time the interior of domain, that belongs to either of the pyramids, is a common cube, one diagonal of which is a segment of the real axis $[-T, T]$. At the same time the interception of two light pyramids results in a figure, built from $(n - 2)$ edges of such cube, excluding the points $-T$ and T. In this case this is a hexagon $ABCDEF$ and it does not belong to the plane $x_1 = 0$, though compiles one of it rectilinear elements.

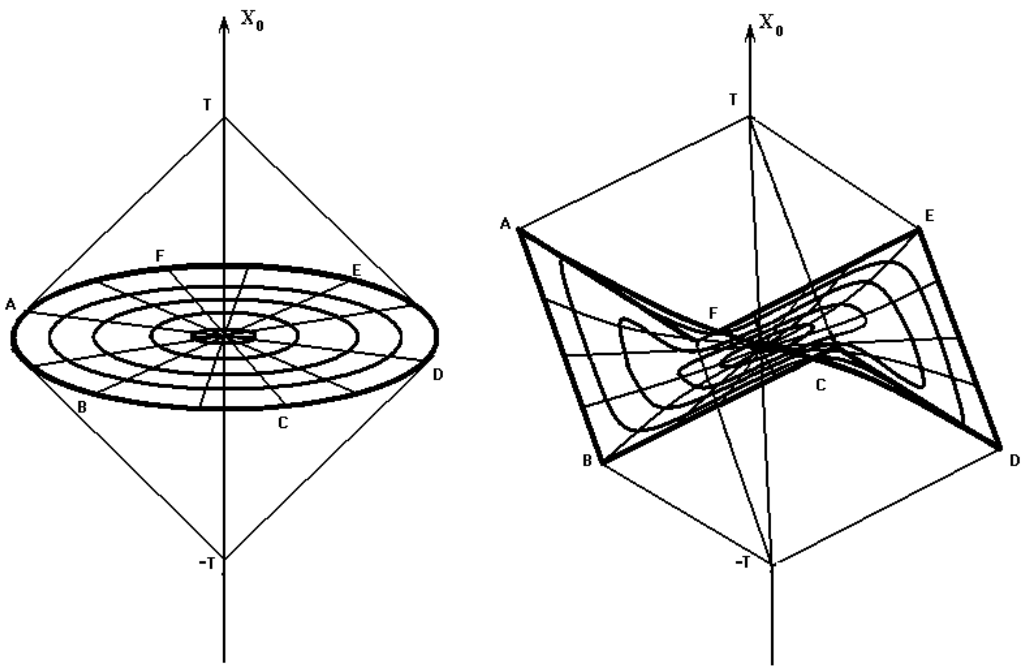

Figure 2: The simultaneous surface of three-dimensional time (right) and in three-dimensional pseudo-Euclidian space (left)

It is analogous in the four-dimensional time: the area that belongs to two facing light pyramids is a four-dimensional cube and the plane of the interception of their isotropic edges is built by 20 2-edges of the cube, that do not include the main diagonal $[-T, T]$.It is difficult to demonstrate this figure using a plane scheme that is why we will limit to the examined above a three-dimensional prototype. In the work [13] there was made an attempt to examine the corresponding dodecahedron (but it seems that the author has lost its principle four-dimensional character and depicted it as a common three-dimensional figure).

In the Minkowskian space the world lines that are parallel to the world line of the observer and touch the figure, which is the interception of two light cones, are accepted as equidistant points of the physical space of the observer, and the value proportional to the axis length of such double cone is referred as the distance. We can act in the analogous way in the four-dimensional time. In this case the parallel to the real axis lines, that come through the point of interception of the edges of two facing light pyramids, become equidistant from it, and in the role of the distant act the value that proportional to the main diagonal of the hypercube that is the result of such interception. In order to find the numerical value of it we should choose 2 real roots from the equalization:

$$x_1^4 - 2(x_2^2 + x_3^2 + x_4^2)x_1^2 + 8(x_2x_3x_4)x_1 + (x_2^4 + x_3^4 + x_4^4 - 2x_2^2x_3^2 - 2x_2^2x_4^2 - 2x_3^2x_4^2) = 0, \quad (11)$$

which are nothing but the abscises of the interception point of the line, which is related to the coordinates x_2, x_3, x_4, and 4 isotropic hyperplanes. One of the roots $x_{1,1}$ corresponds to the point that belong to the pyramid of the past, another $x_{1,2}$ - to the future, as the other 2 redundant roots $x_{1,3}$ and $x_{1,4}$ belong to the edges of the plane of the side pyramids. In this case we can consider the distance to be half of the sum of the first 2 roots: $R_c = 1/2(x_{1,1} + x_{1,2})$, while the index "c" emphasizes that the value is defined by light signals.

The three-dimensional space that appears as the result of such procedure is the Finslerian and is characterized by its indicatrix whose role plays the described above

[13] dodecahedron. The space in its characteristics is quite close to the Euclidean, it comes from the convexity and two-dimensional restraint of its indicatrices, that does not differ greatly from the indicatrix of the Euclidean space, which is a common sphere. But the difference between the Euclidean sphere and the examined dodecahedron is rather principle to mix up their geometries. That is why there was made a conclusion in the work [13] that the idea that in the basis of the geometry of the real macro-world lies the four-dimensional time metrics. But still we think that while making the conclusion one very important circumstance, that when orientating in the real space the observer uses much slower signals rather than the light ones, was not taken into consideration. The light only helps, it is to identify the objects, as the comparison of their distances is realized by other slower means. The fact was not important in the Special Theory of Relativity as the indicatrix of the physical space did not depend on the speed of the signal. It is not like this in the multy-dimensional time. The more the relative speed of the probing signals differs from the light, the less the corresponding indicatrix distinguished from the hyper-plane, the more round become its angles and the more it looks like the three-dimensional sphere. At the limit when the relative speed of signals, with the help of which the physical space is examined, stems to 0, it stops being different from the Euclidean. So if we detect some static objects in the four-dimensional time with the help of the light, and define the distance with the help of other slower signals, so in this case we will come upon only the Euclidean geometry. Let us note that the very condition is complied in the vast majority of common for a man situations.

On the other hand it is not questioned that there is a principle opportunity to carry out an experiment in order to get to know which geometry better suits the real physical space – the Riemannian or the Finslerian. In this case it is important that the distance between fixed objects should be made by other light or slower signals. It is paradoxical but such experiments that do not accept double interpretation lack among the huge number of experimental materials. But the differences that should be traced are not large and that is why can be explained in different ways.

The above accepted conception of building the three-dimensional time explains why in absolutely equal in geometrical rights coordinates of the four-dimensional time the observer, associated with a world line, will register a significant difference between the coordinate that relate to his proper time and the other three. The answer lies in the topological difference between indicatrices of the geometrical and physical spaces. So if the first has the look of a specific 16-line hyperboloid, the second is a ring closed in two dimensions, its right form though depends on the used in measurements signals, is static from topological point of view.

5. Conclusions

Forms that save the scalar form (8), do not leave the intervals invariant, and tot ell the truth are not movements of the four-dimensional time. But as they turn the hyper-planes of the simultaneity (10) into themselves and do not change the three dimensional distances R_c they can act as common physical turns. There can emerge an explanation of the famous paradox - between the forward and rotatory movement. It is difficult to use the principle of relativity to the latter case, and the most famous attempt to examine it was made by Mach, who thought that the centrifugal forces owe their existence to the enormous mass of all the bodies in the Universe. According to Mach if we start turning the whole Universe a static small body will be affected by the centrifugal force that equals the force that emerge during the turning of the body itself. For many people it stays unclear the truth of the statement, and the question itself is still acute. In case we correspond to the real world in place of the Galileo or Pseudo-Euclidean metrics the geometry of the four-dimensional time the problem itself will not appear as the transformation that is responsible for the forward and rotatory movement, correspond to absolutely different continuous symmetries.

The analysis of the multiformity characteristics made in the work that claims to become an alternative to the Minkowski space is far from being finished. But the fact that we can give such condition for one of the most simple Finslerian metrics of the fourth degree that has nothing in common with the usual quadratic form, when it can stimulate not only classical but relative conceptions about the physical space, is worth paying attention to.

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