

From Editorial Board

NUMBER, GEOMETRY AND NATURE

Number is one of the most fundamental concepts not only in mathematics, but in general natural science as well. It may be primary even in comparison with such global categories as time, space, substance, matter, and field. That is why editing the first issue of the journal "Hypercomplex numbers in geometry and physics" the editorial board sincerely hopes that articles not only on numbers in general, but primarily the works that reveal their organic connection with the real world will find here their true scope.

The concept of number in its most general meaning unifies not only common numbers that all of us know from school, but also such objects as the quaternion, the octave, the matrices, etc. Without denying the importance of numbers of all types, let us well emphasize the class chain that has the following shape: natural \rightarrow integer \rightarrow rational \rightarrow real \rightarrow complex. At the same time our aim is to found the possibility of extending the given above classification to numbers of high dimensionality, including those that obey commutative-associative multiplication.

At first sight this plan seems to be absolutely unproductive, for in algebra there exists the Frobenius theorem that claims that multy-component numbers, as being structures subjected to arithmetic properties, end with the complex numbers. At the same time a special stress is laid on the fact that in the according algebras there are no the so-called divisors of zero. Of course, if we take into consideration the real and complex numbers, treating them as the standard, the zero divisor seems to be redundant. Nevertheless, from the point of view of physics and the pseudo-Euclidean geometry closely connected thereto, the zero divisor is one of the most natural objects, for the world lines of the light rays are related to it. The fact that the pseudo-Euclidean planes may be juxtaposed with the algebra of the commutative associative double numbers which have the zero divisors, may serve as the best proof of it. Habitual claims, that the double numbers are too primitive and cannot act as a real competitor to the complex, do not seem to be well-founded, as it would mean in terms of geometry that the Euclidean spaces are more important than the pseudo-Euclidean spaces. Long ago geometricians came to an agreement that both types of space have right to exist; therefore, that is why it is impossible to divide the double numbers as well as the complex numbers proper into the valuable ones and not rather. In our opinion the next conclusion is obvious: in the classification of the value number structures the double numbers should be placed close to the complex ones. If we do treat the double numbers as the fundamentals, then we will not have any argument to keep on ignoring the zero divisors, which means that it is quite possible to create number systems of a larger number of dimensions, and this does not contradict the Frobenius theorem.

The complex quaternions (they are also called biquaternions) are a nice example of such structures. Various interesting works published in the first issue of this journal are devoted to the exploration of the associative complex numbers and not to the ones that are commutative by multiplication. The hope of a success of this trend is based on the fact that the Poincare group, that plays an important role in modern physics, is a subgroup of the full group of continuous symmetries of the eight-dimensional real space of the biquaternions. On the other hand if we accept the fact that the divisor is independent we can build hypercomplex systems, that have commutative-associative multiplication, what has its additional advantages. It is suggested that we should pick them out in a new group of Poly-Numbers to emphasize the special status of such structures.

Lately much attention has not been paid to the exploration of Poly-Numbers, for their structure was commonly considered to be trivial. In a way this is true, but if we put in the first place not algebra but geometry then the multitude increases significantly. It is explained by the fact that spaces (that can be related to Poly-Numbers) as a rule are Finslerian spaces, where some non-linear reflections stand out from linear transformations.

No matter what will be the result of the generalization of the idea of the number, the existence of the Finslerian geometries is the reality, which means that we can explore physics in other or alternative ways. Why not try to change the geometrical basis of physics, and hope that the very geometric basis would be closer to non-quadratic structures, instead of searching hypercomplex structures corresponding to the classical Minkowskian space or to its quadratic modifications. Expecting rather a beautiful and effective confirmation of a close connection between mathematics and physics, we can assume a supposition, that the new geometry must be connected with the most common number structures as the basis of our a little bit risky plan. Here should emerge the Poly-Numbers that on one hand, as it is mentioned above, are quite trivial, but on the other hand are the elements of rather substantial geometries. Even if our expectation will not be fulfilled with Poly Numbers, there is still a vast number of alternatives, and taking into consideration the fundamental nature of the posed problem it is difficult to foresee which of the ways will turn out to be more productive.